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LETTER TO THE EDITOR

One-loop contribution to the dynamical exponents in spin glasses

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Abstract. We evaluate the corrections to the mean-field values of the x and z exponents at the first order in the ϵ-expansion, for $T = T_c$. We find that both x and z are decreasing when the space dimension decreases.

We want to investigate the purely relaxational dynamics of a short-range spin-glass model for $T \to T_c^+$, in the framework of the ϵ-expansion. The dynamical properties of the model in the mean-field theory are very different from those of the models whose dynamical properties are usually investigated in the literature. These new features make the computation of the dynamical critical exponents much more involved than that of the usual models.

In a previous work, [6], we evaluated the Gaussian dynamical fluctuations of the order parameter around the mean-field limit. The aim of this letter is to pursue this analysis, by considering the one-loop correction to the mean-field (MF) theory in a renormalization group calculation. To first order in ϵ, unlike Zippelius [5], we find results that disagree with the conventional Van Hove theory, because we obtain a correction to the kinetic coefficient already to lowest order in the loop expansion. In this work, first we evaluate the correction to the MF value of the critical exponent x that describes the critical slowing down of the dynamical order parameter at the critical point, then we evaluate the correction to the z exponent that describes the critical slowing down of the dynamical spin-glass susceptibility χ_{SG}. Finally, we check that the scaling law, which connects these two exponents, is verified.

We study the soft-spin version of the Edwards–Anderson (EA) model given by the Hamiltonian

$$\beta\mathcal{H} = -\beta \sum_{\langle ij \rangle} J_{ij} s_i s_j + \frac{1}{2} r_0 \sum_i s_i^2 + \frac{1}{4!} g \sum_i s_i^4 \tag{1}$$

where J_{ij} are random Gaussian interactions between the nearest-neighbours sites, with zero average and mean-square fluctuations $[(J_{ij})^2] = j^2/n$ (n is the coordination number). Purely relaxation dynamics is introduced by the Langevin equation

$$\Gamma_0^{-1} \frac{\partial s_i(t)}{\partial t} = -\frac{\partial(\beta\mathcal{H})}{\partial s_i(t)} + \eta_i(t) \tag{2}$$

where η is the usual Gaussian noise with zero average and variance $\langle \eta_i(t)\eta_j(t')\rangle = (2/\Gamma_0)\delta_{ij}\delta(t-t')$. The interesting physical quantities in MF theory are the averaged response and correlation functions, defined respectively by

$$\overline{G}(t-t') = \left[\frac{\partial\langle s_i(t)\rangle_\eta}{\partial h_i(t')}\right]_J \tag{3}$$

$$\overline{C}(t-t') = \left[\langle s_i(t)s_i(t')\rangle_\eta\right]_J \tag{4}$$

where the angular brackets $\langle..\rangle_\eta$ refer to averages over the noise and the square brackets $[..]_J$ over quenched disorder.

Moreover, considering the Gaussian fluctuation, we can define the dynamical spin-glass susceptibility as follows

$$\chi_{\rm SG}(i-j;t_3-t_1,t_2-t_4) = \left[\frac{\partial\langle s_i(t_3)\rangle_\eta}{\partial h_j(t_1)}\frac{\partial\langle s_i(t_2)\rangle_\eta}{\partial h_j(t_4)}\right]_J. \tag{5}$$

The dynamical scaling implies that the decay of $C(t)$ is governed by a characteristic time τ, which diverges at $T_{\rm c}$, as, for long t, we can write

$$C(t) = \frac{1}{t^x}\tilde{q}^+(t/\tau) \tag{6}$$

where $\tilde{q}^+$ is the universal scaling function in the region $T \to T_{\rm c}^+$. The relaxation time divergence, at the critical point, is connected to the correlation length divergence through the dynamical exponent z:

$$\tau \propto \xi^z. \tag{7}$$

According to the scaling hypothesis, this exponent is related to the slowing down of the spin-glass susceptibility, and we expect to have

$$\chi_{\rm SG}(k,\omega) = \omega^{(2-\eta/z)}\tilde{f}(k^z/\omega). \tag{8}$$

The MF behaviour of this model ($n = N$, number of spins, long-range limit), in the critical region, is well known [1–3]. In the low-frequency limit the response and correlation functions are respectively

$$\overline{G}(\omega) = (1-\sqrt{-\mathrm{i}\omega}) \tag{9}$$

$$\overline{C}(\omega) = \frac{2}{\sqrt{\mathrm{i}\omega}+\sqrt{-\mathrm{i}\omega}} \tag{10}$$

while, in the Gaussian approximation, the spin-glass susceptibility is

$$\chi_{\rm SG}(k,\omega_1,\omega_2) = \frac{1}{k^2+\sqrt{-\mathrm{i}\omega_1}+\sqrt{-\mathrm{i}\omega_2}}. \tag{11}$$

The MF value of the dynamical critical exponents x and z, as known, are $1/2$ and 4, respectively.

To deal with Langevin disordered dynamic theory, as usual we use the dynamic functional integral method [7]. In this formalism, it is conventional to introduce an auxiliary field $\hat{s}_i(t)$ and to define an effective Lagrangian of an Hubbard–Stratanovich field $Q_i^{\alpha\beta}(t,t')$, [2, 5], such as

$$2\langle Q_{k=0}^{\alpha\beta}(t_1,t_2)\rangle_{L(Q_{\alpha\beta})} = [\langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\rangle_\eta]_J \tag{12}$$

where the two-component vector field is defined as

$$\phi_i^\alpha = (\mathrm{i}\hat{s}_i, s_i). \tag{13}$$

In considering the correction to the MF approximation we derive the following Lagrangian as series an expansion around the order parameter saddle-point value $\overline{Q^{\alpha\beta}}(t,t')$, [3, 5, 6]:

$$\begin{aligned}L(\delta Q^{\alpha\beta}) = &-\sum_{t_1,t_2}\sum_{i,j}\tilde{K}^{-1}_{i,j}\delta Q_i^{\alpha\beta}(t_1,t_2)A^{\alpha\beta\gamma\delta}\delta Q_j^{\gamma\delta}(t_1,t_2)\\ &+\frac{1}{2}\sum_{t_1,t_2,t_3,t_4}\sum_i \delta Q_i^{\alpha\beta}(t_1,t_2)C^{\alpha\beta\gamma\delta}(t_1,t_2,t_3,t_4)\delta Q_i^{\gamma\delta}(t_3,t_4)\\ &+\frac{1}{3!}\sum_{t_1,t_2,t_3,t_4,t_5,t_6}\sum_i C^{\alpha\beta\gamma\delta\mu\nu}(t_1,t_2,t_3,t_4,t_5,t_6)\delta Q_i^{\alpha\beta}(t_1,t_2)\delta Q_i^{\gamma\delta}(t_3,t_4)\\ &\times\delta Q_i^{\mu\nu}(t_5,t_6).\end{aligned} \tag{14}$$

For the structure and the meaning of each term of (14), the reader is referred to [6]. The non-local connected propagators of the theory are defined as follows:

$$\begin{aligned}G^{\alpha\beta\gamma\delta}(i-j;t_1,t_2,t_3,t_4) &= [\langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\phi_j^\gamma(t_3)\phi_j^\delta(t_4)\rangle_\eta]_J - [\langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\rangle_\eta]_J\\ &\times[\langle\phi_j^\gamma(t_3)\phi_j^\delta(t_4)\rangle_\eta]_J = 4\sum_l(\tilde{K}^{-1})_{il}\sum_k(\tilde{K}^{-1})_{jk}\\ &\times\langle\delta Q_l^{\alpha\beta}(t_1,t_2)\delta Q_k^{\gamma\delta}(t_3,t_4)\rangle_{L(Q_{\alpha\beta})} - 2(\tilde{K}^{-1})_{ij}A^{\alpha\beta\gamma\delta}\delta(1-3)\delta(2-4).\end{aligned} \tag{15}$$

In [6] we evaluated the critical behaviour of these propagators for each combination of the indices $\alpha,\beta,\gamma,\delta$ and in any time interval, when the cubic interactions vanish. We write down the general structure of $G^{2211}(\omega_1,\omega_2,\omega_3,\omega_4)$ which is present in several of the following one-loop functions

$$\begin{aligned}G^{2211}(k;\omega_1,\omega_2,\omega_3,\omega_4) &= [\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)+\delta(\omega_2+\omega_3)\delta(\omega_1+\omega_4)]\\ &\times\tilde{G}^{2211}(k;\omega_1\omega_2)+\delta(\omega_1+\omega_2+\omega_3+\omega_4)\tilde{\tilde{G}}^{2211}(k;\omega_1,\omega_2,\omega_3,\omega_4)\end{aligned} \tag{16}$$

where

$$\tilde{G}^{2211}(k;\omega_1,\omega_2) = \frac{1}{ck^2+\sqrt{-i\omega_1}+\sqrt{-\mathrm{i}\omega_2}} \tag{17}$$

$$\begin{aligned}\tilde{\tilde{G}}^{2211}(k;\omega_1,\omega_2,\omega_3,\omega_4) &= (\overline{C}(\omega_1)\tilde{G}^{2211}(k;-\omega_1,\omega_2)+\overline{C}(\omega_2)\tilde{G}^{2211}(k;\omega_1,-\omega_2))\\ &\times\tilde{G}^{2211}(k;\omega_1,\omega_2)\tilde{G}^{2211}(k;-\omega_3,-\omega_4)g_r(k;\omega_1+\omega_2)\end{aligned} \tag{18}$$

$$g_r(k;\omega_1+\omega_2) = \left(ck^2+\sqrt{-\mathrm{i}(\omega_1+\omega_2)}\right)F_1\left(\frac{k^2}{(\omega_1+\omega_2)^{1/2}}\right) \tag{19}$$

where $F_1(x)$ is an homogenous function of $k^2/\sqrt{\omega}$.

The functional form of the propagators evaluated in [6] is different from that found in [5]. Also the two analytical approaches are different. We have computed, [6], the propagators by performing an expansion in the quartic coupling constant g and resumming the most relevant contributions. In [5], the propagators were evaluated directly in the hard spin limit by assuming some approximations whose consequences can be hardly predicted (e.g. the term $\tilde{\tilde{G}}^{2211}$ is supposed to be zero). It is not surprising that starting from rather different forms of the propagators we obtain different values for the critical exponents. Moreover in [5] there is an inconsistent result: the one-loop contribution to the MF value $\overline{Q^{\alpha\beta}}$ (that is of order $O(\epsilon)$ with respect to $\overline{Q^{\alpha\beta}}$) is found to be vanishing while the correction to the x exponent evaluated by the scaling law $x = d-2+\eta/2z$, where $z = 2(2-\eta)$, is different from zero. Our calculations are not affected by these kinds of problems.

We recall that in this formalism the time-dependent spin-glass susceptibility (5) is

$$\chi_{\rm SG}(i-j;t_3-t_1,t_2-t_4) = [\langle s_i(t_3)\eta_j(t_1)\rangle_\eta \langle s_j(t_2)\eta_i(t_4)\rangle_\eta]_j = \tilde{G}^{1221}(i-j;t_3-t_1,t_2-t_4). \tag{20}$$

Let us consider the one-loop correction to the 'free' theory. We intend to use the propagators derived in [6] to evaluate the contribution of the one-loop Feynman diagrams to the mean value $\overline{Q^{\alpha\beta}}$, to the bare propagators $G^{\alpha\beta\gamma\delta}$, and to the bare cubic vertices.

We consider the one-loop Feynman diagrams as a g-series expansion, by using the correspondent propagators. In analogy with the static case, we can guess that the g-dependent part of the one-loop corrections is not singular at the critical point, as soon as $g \neq 0$. The physical quantities must not be affected by the value of g, provided that it is not zero. The behaviour at $g = 0$ is rather different and the ϵ-expansion starts from $D = 8$, [10].

Concerning the one-point function, let us consider the response function which as a consequence of equation (12) is

$$G(\omega) = 2\langle Q^{21}_{k=0}(\omega)\rangle. \tag{21}$$

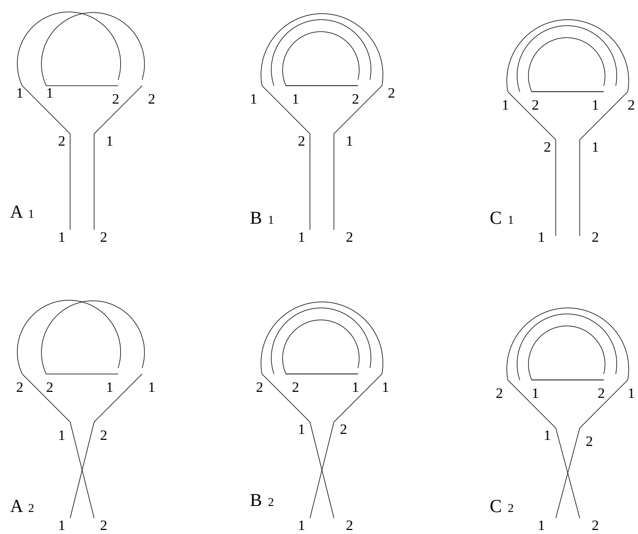

Figure 1. One-loop diagrams that contribute to the x exponent.

The diagrams that occur in one-loop correction to the MF value of the x exponent are shown in figure 1: two continuous lines represent a bare propagator factorized in time

$$\tilde{G}^{\alpha\beta\gamma\delta}(k;w_1,w_2)[\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)+\delta(\omega_1+\omega_4)\delta(\omega_2+\omega_3)]$$

three lines the bare propagator connected in time

$$\tilde{\tilde{G}}^{\alpha\beta\gamma\delta}(k;w_1,w_2,\omega_3,\omega_4)\delta(\omega_1+\omega_2+\omega_3+\omega_4)$$

and, finally, the triangle in the centre of the diagrams is a cubic vertex factorized in time. We succeed in evaluating the singular behaviour of the diagrams A_1 and A_2, while we have to use a trick to take into account the contribution from the others. By using the series expansion in g of the propagators in the loop, see [6], we obtain a correspondent g series of the two one-particle irreducible (1P.I.) diagrams, but the zero term is missing. Let us add and subtract the term we need. As in the static case (where we deal with just the momentum variable k, and we easily manage to resum the g-series), we can suppose that the resummation of the series gives a non-singular behaviour at the critical point (the presence of g removes the pole at zero momentum for $T = T_c$). We are left with the diagrams A_1, A_2 and with the two diagrams of zero order in g that we need for the series resummation (with negative sign) (see figure 2):

$$\left(\frac{u}{2}\right) 2 \left[\int \frac{d^d k}{(2\pi)^d} \frac{1}{ck^2 + 2\sqrt{i\omega}} - 2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{ck^2 + \sqrt{i\omega}} \right]. \tag{22}$$

The propagators involved in these 'new' diagrams are represented in figure 2 with two crossed lines. By evaluating the previous integrals for $d = 6$, we find that, to first order in the loop expansion, the response function is

$$\begin{aligned} G(\omega) &= 1 - (-i\omega)^x = \overline{G}(\omega) + 2u\langle \delta Q^{21}(\omega) \rangle \\ &= 1 - \sqrt{-i\omega} - \frac{k_6}{(2\pi)^6} u^2 \left[I(\omega = 0) + \frac{1}{4}\sqrt{-i\omega}\ln(\omega) \right]. \end{aligned} \tag{23}$$

As for the static case u has been introduced as the expansion parameter in the cubic vertices, to apply the renormalization group method for critical phenomena. The factor u^2 in equation (23) is due to the fact that the saddle-point response function (9) is of order $(1/u)$.

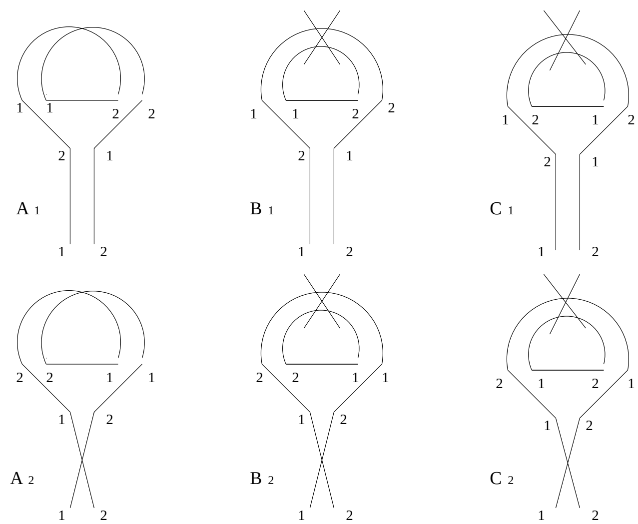

Figure 2. g-series resummation of the one-loop diagram contribution to the x exponent.

In the same way, we derive the flux equation of the 1P.I. vertex functions which allow us to determine the fixed point below six dimensions. As for the conventional Langevin dynamical theories, the IR stable fixed point below six dimensions is the same as that for the static case (i.e. we find the same relevant diagrams):

$$(u^*)^2 = \frac{(2\pi)^6}{k_6} \frac{\epsilon}{2}. \tag{24}$$

The series expansion in ϵ for the static exponents is evaluated to third order in [8] and [9].

Let us determine the first-order correction for the dynamical case. Substituting the fixed point value (24) into (23) we obtain for x

$$x = \frac{1}{2}\left(1 - \frac{\epsilon}{4}\right). \tag{25}$$

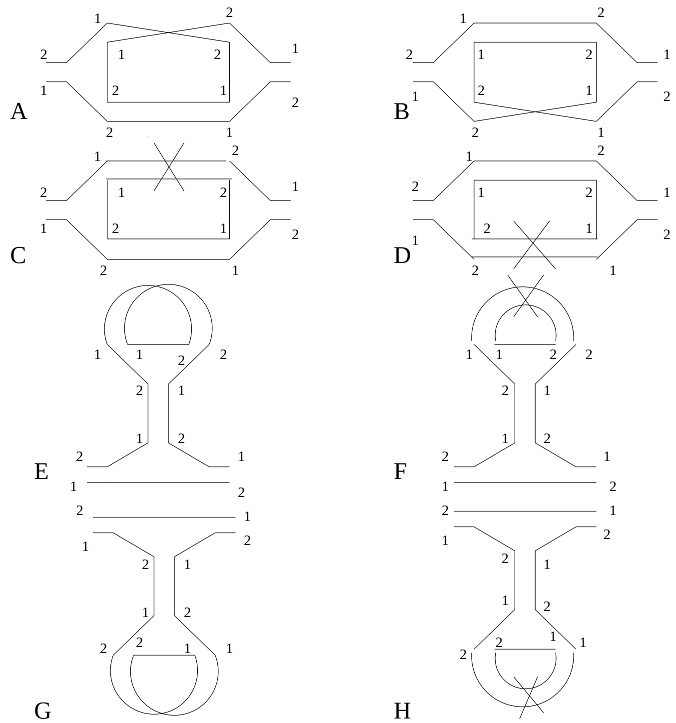

Figure 3. g-series resummation of the one-loop diagram contribution to the self-energy.

In the same way, we find that the relevant one-loop 1P.I. diagrams for the self-energy $\Sigma(\omega_1, \omega_2)$, shown in figure 3, give the following contribution to the spin-glass susceptibility (20):

$$\text{A}: \quad \frac{u^2}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + 2\sqrt{\mathrm{i}\omega_1}} \frac{1}{ck^2 + \sqrt{\mathrm{i}\omega_1} + \sqrt{-\mathrm{i}\omega_2}} \right) \tag{26}$$

$$\text{B}: \quad \frac{u^2}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + 2\sqrt{-\mathrm{i}\omega_2}} \frac{1}{ck^2 + \sqrt{\mathrm{i}\omega_1} + \sqrt{-\mathrm{i}\omega_2}} \right) \tag{27}$$

$$\text{C}: \quad -2\frac{u^2}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + \sqrt{\mathrm{i}\omega_1}} \frac{1}{ck^2 + \sqrt{-\mathrm{i}\omega_2}} \right) \tag{28}$$

$$\text{D}: \quad -2\frac{u^2}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + \sqrt{-\mathrm{i}\omega_2}} \frac{1}{ck^2 + \sqrt{\mathrm{i}\omega_1}} \right) \tag{29}$$

$$\text{E}: \quad \frac{u^2}{2} 2 \frac{1}{cp^2 + 2\sqrt{\mathrm{i}\omega_1}} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{ck^2 + 2\sqrt{\mathrm{i}\omega_1}} \tag{30}$$

$$\text{F}: \quad -2\frac{u^2}{2} 2 \frac{1}{cp^2 + 2\sqrt{\mathrm{i}\omega_1}} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{ck^2 + \sqrt{\mathrm{i}\omega_1}} \tag{31}$$

$$\text{G}: \quad \frac{u^2}{2} 2 \frac{1}{cp^2 + 2\sqrt{-\mathrm{i}\omega_2}} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{ck^2 + 2\sqrt{-\mathrm{i}\omega_2}} \tag{32}$$

$$\text{H}: \quad -2\frac{u^2}{2} 2 \frac{1}{cp^2 + 2\sqrt{-\mathrm{i}\omega_2}} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{ck^2 + \sqrt{-\mathrm{i}\omega_2}}. \tag{33}$$

To first order in ϵ, the sum of the contributions to the z exponent of the diagrams A, B, C and D vanishes. On the other hand, from the sum of the diagrams E, F, G and H we obtain the following contribution to the 1P.I. two-point function:

$$\chi_{SG}^{-1}(p; -\omega_1, \omega_2)\big|_{p=0} = \overline{\chi_{SG}^{-1}}(-\omega_1, \omega_2) + \Sigma(\omega_1, \omega_2)$$
$$= \sqrt{i\omega_1} + \sqrt{-i\omega_2} + \frac{u^2}{4}\frac{k_6}{(2\pi)^6}\left(\sqrt{i\omega_1}\ln(\omega_1) + \sqrt{-i\omega_2}\ln(\omega_2)\right). \tag{34}$$

At the fixed point u^* given from (24), we obtain the following correction to the z exponent:

$$z = \frac{(2-\eta)}{\frac{1}{2}\left(1+\frac{1}{4}\epsilon\right)} = 4\left(1 - \frac{\epsilon}{12}\right). \tag{35}$$

The scaling relation between the exponents x and z, which we recall to be

$$x = \frac{d-2+\eta}{2z} \tag{36}$$

is verified, to first order in ϵ.

Numerical simulations for the exponents x and z can be found in the literature, the values are $z \approx 7$ and $x \approx 0.06$ in dimensions $D = 3$, [11], and $z \approx 5$ and $x \approx 0.15$ in dimensions $D = 4$, [12]. Our prediction states that both the values of x and z are decreasing when the dimension decreases. This is true for x but not for z. The apparent discrepancy that we have with the behaviour of z should not worry us. In fact, also in the static case, the critical exponents for spin glasses have a badly convergent ϵ-expansion and the prediction of this expansion can be hardly applied in three or four dimensions.

A numerical study of what happens in five dimensions is necessary. Moreover, it should be noted that usual arguments imply that our computation predicts, without ambiguities, that the logarithmic corrections in six dimensions are such to decrease the effective value of the exponents.

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